Abstract of the Thesis

A STUDY OF MULTIWAVELET PACKETS

By

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Multiwavelet packet analysis is an important generalization of multiwavelet analysis. Multiwavelet packet functions comprise a rich family of building block functions and are still localized in time, but offer more flexibility than multiwavelets in representing different types of signals. In particular, multiwavelet packets are better at representing signals that exhibit oscillatory or periodic behavior. Multiwavelet packets are superpositions of multiwavelets and they form bases which retain many of the orthogonality, smoothness, symmetry, vanishing moments and localization properties of their parent multiwavelets. Multiwavelet packets are organized naturally into collections, and each collection is an orthogonal basis for $L^2(R^d)$. It is a simple, but very powerful extension of multiwavelets and multiresolution analysis (MRA) of multiplicity L. The multiwavelet packets allow more flexibility in adapting the basis to the frequency contents of a signal and it is easy to develop a fast multiwavelet packet transform.

Multiwavelet packet functions are generated by scaling and translating a family of basic function shapes, which include scaling vector φ and multiwavelet ψ. In addition to φ and ψ there is a whole range of multiwavelet packet functions $\mathbf{W}_l^{\,n}$. These functions are parameterized by an oscillation or frequency index n. A scaling vector corresponds to n = 0, so $\varphi = \mathbf{W}_l^{\,0}$. A multiwavelet corresponds to n = 1, so $\psi = \mathbf{W}_l^{\,1}$. Larger values of n correspond to multiwavelet packets with more oscillations and higher frequency.

Characterization of wavelets with dilation factor 2 has been studied by many authors in different directions. A complete characterization of wavelets in the Fourier transform domain was given by Gripenberg and independently by Wang. The work of Gripenberg and Wang was extended to higher dimensions for integer expanding dilation matrices by Frazier et al. , Han, Ron and Shen and Calogero. **In Chapter-II**, we have studied the characterization of all orthonormal multiwavelet packets on general lattices and this result generalizes the result of Han, Ron and Shen, and Calogero about multiwavelets.

In Chapter-III, we have studied some new characterizations of orthonormal multiwavelet packets associated with integer dilation matrices by means of basic equations in Fourier domain. Our approach is based on the fundamental work of Bownik who has given a complete characterization of orthonormal multiwavelets based on the quasi affine frames and dual Gramians.

In Chapter-IV, we study the characterization of dual multiwavelet packets over integer dilation matrices and we have shown the necessary and sufficient condition for the functions $\mathbf{W}_l^{\,n}$ and $\mathbf{W}_l^{\,n\,\sim}$ in $L^2(R^d)$ to constitute a dual pair for $L^2(R^d)$. Moreover,

as a particular case, when $\mathbf{W}_1^n = \mathbf{W}_1^{n\sim}$, we obtain a complete characterization of all orthonormal multiwavelet packets in $L^2(R^d)$.

In 1999, Guido Weiss conjectured during his "wavelet seminar" at Washington University that dilation parameter of wavelets can be extended to arbitrary dilation factor $a > 1$. The conjecture was soon proved by Chui and Shi for an arbitrary dilation factor $a > 1$ and they have established a complete characterization of orthonormal wavelets. Soon after, another two participants in the seminar, Bownik and Rzeszotnik proved Weiss conjecture for integer dilation factors $a \geq 2$ for multiwavelets. Chui and his colleagues extended this result to higher dimensions for arbitrary dilation matrices. **In Chapter-V**, we study some new characterizations of dual multiwavelet packets associated with arbitrary (expanding) dilation matrices and have also discussed some special cases of dilation matrices. Our results are the generalization of the results of Bownik, Rzeszotnik, Calogero and Chui et al.