Abstract of the Ph.D. thesis

entitled

ON INTUITIONISTIC FUZZY DISCRETE STRUCTURES

submitted by

by

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The work in this thesis is on the applications of fuzzy set theory of Zadeh [125] and intuitionistic fuzzy set theory of Atanassov [2] in the theory of fuzzy algebra.

It is asserted that the most important and potential generalization of fuzzy sets came in the form of Intuitionistic Fuzzy Sets(IFS) developed by Atanassov. Recently Gau and Buehrer reported in IEEE [45] the theory of vague sets. *But vague sets and intuitionistic fuzzy sets are same concepts as clearly justified by Bustince and Burillo in [23]. Consequently, in this thesis the two terminologies 'vague set' and 'intuitionistic fuzzy set' have been used with same meaning and objectives.*

The actual contribution in this thesis is in the areas of 'theory of intuitionistic fuzzy groups', and 'intuitionistic fuzzy relations'. A rigorous study on the theory of intuitionistic fuzzy groups have been done, several propositions are proved with examples. Then the notion of intuitionistic fuzzy normal groups and intuitionistic fuzzy charactersistic groups is introduced, and their properties are studied. Finally the notion of intuitionistic fuzzy relations is introduced. The concept of composition of intuitionistic fuzzy relations is given, and its potential scope of possible application in many real life problems is justified by presenting a simple example of everyday problem.

Group theory plays an important role in many areas of Physics, Communication, Quantum Mechanics, etc. Since the inception of the theory of Fuzzy Groups by Rosenfeld, a huge number of works have been reported in this area. Theory of fuzzy relations has been playing a great and significant role in different areas in Computer Science, in Management Science, in Medical Science, in Banking and Finance, in Social Sciences, and in many more areas. Consequently it is

expected that the work reported in this thesis will add an element of support to the existing theory of fuzzy algebra and to the theory of mathematical relations. Precision assumes that the parameters of a model represent exactly either our perception of the phenomenon modelled or the features of the real system that has been modelled. Generally precision indicates that the model is unequivocal, that is, it contains no ambiguities. By crisp we mean yes-or-no type rather than more-or-less type. In conventional dual logic, for instance, a statement can be true or false-and definitely nothing in between. Vagueness, imprecision and uncertainty have so far been modelled by classical set-theoretic approach. According to this approach, borderline elements can be either put into the set or should be kept outside it. Hence it becomes inadequate for applying to humanistic type of problems. Zadeh [125], in 1965 initiated the notion of fuzzy set theory as a modification of the ordinary set theory, which turned out to be of far reaching implications. Vague notions can be modelled using this theory.

A fuzzy set is a class of objects in which the transition form membership to non-membership is gradual rather than abrupt. Such a class is characterized by a membership function which assigns to an element a grade or degree of membership between 0 and 1. For a beginner on fuzzy set theory, the work in [40], [57], [61], [62], [119], [132], [137-141], etc. are good enough to start with. If we look at the developmental history of mathematical systems or structures, we see that a mathematical system is, in general, suggested by situations which, while they are different, have some basic features in common so that the emergence of a mathematical system is essentially the result of a process of unification and abstraction. A mathematical system, thus, lays bare the structurally essential relations between otherwise distinct entities. So, it may be accepted that the results of the study of a mathematical system will be valid for each of those otherwise different situations which provided motivation and inspiration for the same. Such a study also provided an economy of effort and leads to a better and fuller understanding of the motivation situations. Even without considering the motivation situations inherent in cybernetics and general systems prevailing in the emerging man-machine civilization, if we just consider

everyday language, we see that we are concerned with statements which are often distinguished as interrogative, imperative, exclamatory or declarative. In classical mathematical systems, we deal with only those statements which are declarative in nature and which may be either true or false. Fuzzy mathematical systems, whose foundation was laid by Zadeh [125] in his classic papers on the theory of fuzzy subsets and the theory of possibility deal with situations of interrogative, imperative, exclamatory and also declarative statements. So, even in an intuitive sense, fuzzy set theory is a generalization of classical set theory, which has been algebraically established in several of the above-mentioned sources. As a result, one of the most important motivations and one of the main aims of most of the research in fuzzy set theory, is to furnish mathematical models which are able to describe systems or classes of systems which elude traditional analysis. They are being attempted in two different, non-contradictory approaches—using well-known mathematics, interpreting them in a different way and trying to build a new methodology of the modeling of reality in its complexity or by making abstractions which are weaker than the usual ones and based ultimately on a conceptual point of view even if not on a formal one. Different authors from time to time have made a number of generalizations of fuzzy set theory of Zadeh [125] with different objectives ([2], [23], [42], [45], [51], [63], [74], [127], [134]). For each such generalization, one (or more) extra edge is added with the fuzzy theory with specialized type of aim and objective. Thus, a number of higher order fuzzy sets are now in literatures and are being applied into the corresponding more specialized application domains. While fuzzy sets are applicable to each of such application domains, higher order fuzzy sets can not, because of their specialization in character by birth. Application of higher order fuzzy sets makes the solution-procedure more complex, but if the complexity on computation-time, computation-volume or memory-space are not the matter of concern then a better results could be achieved.

Out of many higher order fuzzy sets, the notion of intuitionistic fuzzy set theory (IFS theory) introduced by Atanassov [2] is of interest to us. All fuzzy sets can be viewed as intuitionistic fuzzy sets, but the converse is not true. Many authors

have asserted that there are a large number of real life problems for which IFS theory is a more suitable tool than fuzzy set theory. The work in this thesis is on the applications of fuzzy set theory of Zadeh [125] and intuitionistic fuzzy set theory of Atanassov [2] in algebra and relations.

The organization of the thesis is as below:-

(1) A general introduction of the complete work of the thesis is introduced in Chapter-1.

(2) In Chapter-2, we present basic preliminaries on Zadeh's theory of fuzzy sets, different operations on them, theory of higher order fuzzy sets specially intuitionistic fuzzy sets and their properties.

(3) In Chapter-3, we present basic preliminaries on theory of higher order fuzzy sets specially intuitionistic fuzzy sets and their properties.

(4) Since the inception of the theory of fuzzy algebra by Rosenfeld, a good amount of work have been reported so far by various authors in this area. The intuitionistic fuzzy sets have an extra edge over fuzzy sets. Consequently there is a reason that intuitionistic fuzzy algebra will play a significant role in the area of fuzzy algebra in due time. In Chapter-4 we study the notion of intuitionistic fuzzy groups, their various properties and make some characterizations.

(5) In chapter-5, the notion of intuitionistic fuzzy normal groups and intuitionistic fuzzy characteristic groups are introduced, and their properties are studied.

(6) Fuzzy relations have a wide range of applications ([13], [17-22], [26], [27], [48], [52], [73], [97-99], [101-106], [111], [126]) in different areas in Computer Science specially in DBMS, in relational database of Codd's model (see [35], [95], [114]), in Management Science, in Medical Science, in Banking and Finance, in Social Sciences etc. In Chapter-6, we study the notion of intuitionistic fuzzy relations and different operations on them. We present an useful application of