ABSTRACT OF Ph.D. THESIS

Title of Ph.D. Thesis: *Certain Investigations in the Field of Multiple Gaussian Hypergeometric Functions.*

Research Scholar: *Rahul Singh.*

Research Supervisor: *Prof. Mohammad Idris Qureshi.*

Department of Applied Sciences & Humanities, Faculty of Engineering & Technology, Jamia Millia Islamia, New Delhi-110025, India.

The present thesis comprises of SEVEN CHAPTERS. A brief summary of the problems is presented at the beginning of each chapter and then each chapter is divided into a number of sections. Equations in every section are numbered separately. For example, in the small parentheses (a.b.c) the last figure denotes the equation number, the middle one the section and the first indicates the chapter to which it belongs. Sections, Articles, Definitions and Equations have been numbered chapter wise.

The aim of the CHAPTER 1, is to introduce several classes of special functions, which occur rather frequently in the subsequent chapters. In this chapter, different forms of gamma function has been discussed. Legendre duplication and triplication formulae; Psi function; Polygamma function $\psi^{(m)}(x)$; Complete and Incomplete Beta and gamma functions; Fractional derivative; Hankel's contour integral formula; a useful limit formula for certain infinite products; Pochhammer symbol; Ordinary hypergeometric function of one variable ${}_AF_B$, its convergence conditions, properties associated with well-poised series, very well-poised series, Saalschützian series, nearly poised series of first and second kinds; Wright's generalized hypergeometric function ${}_p\Psi_q,\ {}_p\Psi_q^\star$; Hypergeometric summation theorems and Reduction formulae; Catalan's constant G; Kampé de Fériet's double hypergeometric function; Multiple ordinary hypergeometric function of Srivastava−Daoust and Some series identities, etc.

It provides a systematic introduction to most of the important special functions that commonly arise in practice and describes many of their

salient properties. This chapter is intended to make the thesis as self contained as possible.

In CHAPTER 2, we obtain four finite sums of Exton's double hypergeometric polynomials G in terms of generalized hypergeometric polynomials of one variable, by the applications of finite series identities of Srivastava. Also in the present chapter, we have obtained some interesting finite summation formulae (not recorded earlier) involving Exton's double hypergeometric function H by series rearrangement technique.

Further, we obtain some interesting finite sums of general triple hypergeometric series $F^{(3)}$ of Srivastava in terms of general double hypergeometric series G of Exton by series rearrangement technique.

In CHAPTER 3, we obtain some new hypergeometric transformations of $D_1,\ D_2,\ D_3,\ D_4$ and K_{13} into $K_3,\ K_6,\ K_7,\ K_{11}$ and G_B respectively. Two erroneous transformations of Exton are also corrected here.

The main object of CHAPTER 4 is to generalize Ramanujan's integral (obtained with the help of his Master theorem). Motivated by the works of Mridula Garg and Shweta Mittal, we evaluate some definite integrals associated with Gaussian hypergeometric function ${}_AF_B$, Wright's generalized hypergeometric functions ${}_p\Psi_q,\ {}_p\Psi_q^*$. An integral of Prof. R. P. Agarwal associated with Ramanujan's result, is also modified here.

The main object of CHAPTER 5 is to determine two multiplication formulae for double hypergeometric functions of Exton and Kampé de Fériet, by the application of Rainville's theorem on generating function.

In CHAPTER 6, we obtain some generating relations involving Exton's double hypergeometric functions denoted by H and G which are the generalizations and unifications of Kampé de Fériet's double hypergeometric functions, another double hypergeometric function of Exton denoted by X and some double hypergeometric functions of Horn. Some known generating relations scattered in the literature are also obtained as special cases.

In CHAPTER 7, we obtain seven general multiple series identities and Gaussian hypergeometric reduction formulae for Srivastava-Daoust double hypergeometric functions.

Several formulae were given by C. F. Gauss(1812-1866), E. E. Kummer(1836) and W. N. Bailey(1928-1959) expressing the product of two hypergeometric series as a hypergeometric series, such as $e^{-x}\ {}_1F_1(x)$ as a series of the type ${}_1F_1(-x)$ and $(1+x)^{-p}\ {}_2F_1\left(\frac{4x}{(1+x)^2}\right)$ as a series of the type ${}_2F_1(x)$. Another theorem of a similar type, which was given by S. Ramanujan(1911-1919), expressed ${}_1F_1(\alpha;\rho;x)$, ${}_1F_1(\alpha;\rho;-x)$ as a series of the type ${}_2F_3(\frac{x^2}{4})$. This result was proved by C. T. Preece(1923-1924), who obtained other results of a similar kind from the consideration of differential equations. F. J. W. Whipple(1925-1937) obtained a formula expressing $(1-x)^{-p}\ {}_3F_2\left(\frac{-4x}{(1-x)^2}\right)$ as a hypergeometric series of the type ${}_3F_2(x)$. In this chapter a system atic search is made for such formulae. Evidently if the product of two hypergeometric series can be expressed as a hypergeometric series with argument x, the coefficient of x^n in the product must be expressible in terms of Gamma functions.

Similar results also were given by A. Cayley (1858), T. Clausen (1828), W. McF. Orr(1899), G. N. Watson(1906-1959), A. C. Dixon(1891-1905), G. H. Hardy(1920-1923) and L. J. Slater(1951-1966).

The present chapter contains seven general multiple series identities which extend and generalize the theorems.

A detailed bibliography appears at the end; with the author names in alphabetical order. References to the bibliography are numbered. The thesis includes appendices which contain reprints of a few published papers and Gamma tables, etc.