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ABSTRACT

Special functions are the solutions for a wide class of mathematically and physically relevant functional equations. In addition, special functions play an important role in mathematical physics. The special functions of mathematical physics appear often in solving partial differential equations (e.g. in the method of separation of variables) or in finding eigen functions of differential operators (particularly in certain curvilinear systems of co-ordinates).

The purpose of the present research is to study generalized functions, which can provide a unification scheme for the well known special functions (scattered in the existing literature). Especially the generalization of the special functions which are widely used in various areas of applied mathematics have been dealt with. The results thus established do not merely generalize the results given earlier by various workers but also yield a number of new results. The findings can be applied to physics, statistics, engineering and other branches of science.

The present thesis comprises of EIGHT CHAPTERS as follows:

The aim of the CHAPTER 1, is to introduce several classes of special functions, which occur rather frequently in the subsequent chapters. It provides a systematic introduction to most of the important special functions that commonly arise in practice and describes many of their salient properties. This chapter is intended to make the thesis as self contained as possible.

In CHAPTER 2, we obtain two Gaussian hypergeometric summation theorems, general double series identities and various combinations of hypergeometric forms of $e^{ax}\sin bx$, $e^{ax}\cos bx$, $e^{ax}\sin(bx+c)$ and $e^{ax}\cos(bx+c)$.

The main object of CHAPTER 3 is to generalize and unify the results of Agarwal, Carlitz, Maier, Searle, Slater and Verma, on truncated unilateral and non terminating ordinary generalized hypergeometric series, by using series iteration technique and theory of polynomial equations.

CHAPTER 4, is motivated by the works of L. J. Slater and A. Verma. In this chapter we have derived some results on truncated unilateral generalized hypergeometric series of positive unit argument subject to certain conditions in numerator and denominator parameters.

The main object of CHAPTER 5 is to establish some summation theorems for truncated unilateral generalized hypergeometric series associated with negative unit

argument. Applying Rainville's limit formula for certain infinite products, some non terminating hypergeometric summation theorems with negative unit argument are also deduced, in terms of Gamma functions subject to certain conditions.

In CHAPTER 6, Motivated by the evaluation of indefinite integrals of $\sin(ax^2 + 2bx + c)$, $\cos(ax^2 + 2bx + c)$ and $\exp(-ax^2 - 2bx - c)$ in terms of Fresnel's integrals, error function, complementary error function and probability integral; we obtain some indefinite integrals of the product of polynomial function and generalized hypergeometric function ${}_AF_B$ (whose argument is another polynomial function) in terms of multivariable hypergeometric function of Srivastava-Daoust. Making suitable adjustments of parameters and variables in our indefinite integrals and using hypergeometric forms of special functions and elementary functions, we can find a number of known and unknown indefinite integrals of transcendental functions and special functions.

In CHAPTER 7 we obtain some results on truncated bilateral hypergeometric series of positive unit argument, have been derived by using series rearrangement technique and theory of polynomial equations, subject to certain conditions.

The main object of CHAPTER 8 is to establish some summation theorems for truncated bilateral generalized hypergeometric series involving negative unit argument given by

$$ {}_BH_B[g_1,\, g_2,\, \ldots,\, g_B\ ;\, 1+h_1,\, 1+h_2,\, \ldots,\, 1+h_B\ ;\ -1]_{2P-\delta}^{2R-\theta}, $$

$$ {}_{B+1}H_{B+1}[g_1,\, g_2,\, \ldots,\, g_B,\, 1-\varepsilon\ ;\, 1+h_1,\, 1+h_2,\, \ldots,\, 1+h_B,\, -\varepsilon\ ;\ -1]_{2P-\delta}^{2R-\theta}, $$

$$ {}_{B+2}H_{B+2}[g_1,\, g_2,\, \ldots,\, g_B,\, 1-\varpi,\, 1-\varrho\ ;\, 1+h_1,\, 1+h_2,\, \ldots,\, 1+h_B,\, -\varpi,\, -\varrho\ ;\ -1]_{2P-\delta}^{2R-\theta}, $$

$$ {}_{B+3}H_{B+3}[g_1,\, g_2,\, \ldots,\, g_B,\, 1-\varphi,\, 1-\upsilon,\, 1-\eta\, ;1+h_1,\, 1+h_2,\, \ldots,\, 1+h_B,\, -\varphi,\, -\upsilon,\, -\eta\, ;-1]_{2P-\delta}^{2R-\theta}, $$

$$ {}_{B+E}H_{B+E}[g_1,\, \ldots,\, g_B,\, 1-\Xi_1,\, \ldots,\, 1-\Xi_E\ ;\, 1+h_1,\, \ldots,\, 1+h_B,\, -\Xi_1,\, \ldots,\, -\Xi_E\ ;\ -1]_{2P-\delta}^{2R-\theta} $$

and

$$ {}_{2B}H_{2B}[g_1,\, \ldots,\, g_B,\, 1-\Lambda_1,\, \ldots,\, 1-\Lambda_B\ ;\, 1+h_1,\, \ldots,\, 1+h_B,\, -\Lambda_1,\, \ldots,\, -\Lambda_B\ ;\ -1]_{2P-\delta}^{2R-\theta}, $$

using series iteration techniques; where $\varepsilon, \varpi, \varrho, \varphi, \upsilon, \eta, \Xi_E$ and Λ_B are the functions of parameters $g_1, g_2, \ldots, g_B, h_1, h_2, \ldots, h_B$. Applying Rainville's limit formula for certain infinite products, some non terminating bilateral hypergeometric summation theorems with negative unit argument are also deduced, in terms of Gamma functions subject to certain conditions.

A detailed bibliography appears at the end; with the author names in alphabetical order.