

Abstract of Ph. D. Thesis

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Title of the Thesis : A Study of Different Available Mathematical Models and their Effectiveness Relative to Black-Scholes Model in the Present Scenario with New Recommendations

The thesis describes how the confluence of mathematics, finance, economics, and computation technology can prove to be so effective in the practice of Financial Engineering. The tool is being used exponentially in portfolio management and for hedging purposes and thus, strong efforts have been projected on modeling of derivative products. This thesis addresses one of the major problems of pricing European plain vanilla options namely the constant volatility assumption of the Black-Scholes model concentrating the volatility of the underlying asset.

Rigorous efforts are being made since the four decades to root out the problem of pricing options with non-constant volatility. The suggested models can be positively categorized into deterministic volatility models and stochastic volatility models. In most of the cases of the previous researches on this issue have been tried with comparing and contrasting Black-Scholes model against the various models crafted by the researches. This study, in this way also sounds unique because for the first time the various models of option pricing have directly been analyzed upon the live data of S&P CNX Nifty index options procured right from the market and instead of comparing it with the benchmark Black-Scholes model we have made a fresh approach to examine it with market.

This thesis investigates to find out an apt model which can completely describe the Nifty Index options. All the models are estimated and compared for out-of sample relative pricing performance.

The thesis is organized as follow: **Chapter-1** gives the introduction and overall development of financial mathematics. This chapter further explains basic concepts of mathematical finance and classical Black-Scholes model, which is the focus of present thesis.

Chapter - 2 Focuses on the deterministic volatility models which define volatility as a deterministic function of asset price and time. At the end, we have defined the Practitioner Black-Scholes model of Dumas et. al. incorporating parabolic smile/smirk shape of the implied volatility. Gram-Charliear model of Backus et. al. that accounts non-lognormal skewness and kurtosis of index return time series and CEV model of Cox and Ross which incorporates correlation of index return and volatility.

Chapter - 3 Analyzes stochastic volatility models retaining enough analytical tractability namely Hull and White, Heston and the Heston Nandi GARCH models.

Chapter - 4 Explains the data screening procedure, option categories, basic characteristics of S&P CNX Nifty 50 Index options and empirical techniques employed in testing option pricing models. A cross-sectional analysis based on moneyness and maturity is done for evaluating the forecasting performance of option pricing models and to find the most efficient model for pricing S&P CNX Nifty index options, across moneyness-maturity groups.

Chapter - 5 Endeavor to investigate empirically the out-of-sample moneyness-maturity forecasting performance of deterministic and stochastic volatility option pricing models to determine the better alternate to minimize the price bias between market and model price.

Chapter - 6 The last chapter of this thesis which discloses the major finding of the thesis along with new recommendations. Based on techniques employed we found strong evidence that Nifty daily returns can be characterized by the GARCH models. We finally conclude that none of the models is completely reproducing the market prices as all of them are producing errors relative to the market. Judging the models on the parameters of internal consistencies we found that all models are unable to measure correctly, but simultaneously we also found that pricing performance of PBS and Heston model is far better than the classical Black-Scholes model (model reluctant to volatility). Moreover, we have not found a single model which dominates the rest in pricing Nifty index options across moneyness and maturity.