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ABSTRACT

The present thesis is some contributions to the ideal theory in different algebraic structures, viz, prime ideals of b-semirings, weakly prime ideals of involution ordered $\Gamma-$semigroups, bi-ideals of ordered Γ-semigroups and bi-ideals of LA-Γ-semigroups, quasi-ideals and bi-ideals of regular and intra-regular ordered Γ-semigroups and quasi-ideals of abstract affine Γ-nearrings.

In **chapter one**, some basic definitions which are needed to prove the results in the subsequent chapters have been stated. It serves the purpose to acquaint the reader with the terminology and basic facts often used to make the thesis as much self contained as possible.

Chapter two deals with prime ideals of b-semirings. The concept of prime ideal plays an important role in the theory of commutative rings. But almost nothing is studied on prime ideals of b-semirings, and one of the goals of the present chapter is to contribute to the theory of b-semirings and to highlight that the subject is more stimulating than one would possibly think. In this chapter, we characterize b-semirings through prime right ideals, prime ideals, completely prime ideals, irreducible prime ideals and strongly irreducible prime ideals. Finally, we introduce and study fully prime right b-semirings and right weakly regular b-semirings. The contents of this chapter are partially published in **International Mathematical Forum 22(2012)**, **1065-1069(Bulgaria)** and the rest of this chapter is communicated to **Novi Sad Journal of Mathematics (Serbia)** which is under second revision.

In **Chapter three**, we introduce involution in ordered Γ-semigroups. Then we generalize results by G. Szasz, N. H. McCoy, O. Steinfeld and M. Petrich on weakly prime ideals to involution ordered Γ-semigroups. It is proved that in the involution ordered Γ-semigroup S such that the involution preserves the order, an ideal of S is prime if and only if it is both weakly prime and semiprime and if S is commutative, then the prime and weakly prime ideals of S coincide. We also prove a number of subsidiary results, and most notably we

prove that if S is an ordered Γ-semigroup with order preserving involution, then the ideals of S are prime if and only if S is intra-regular. The entire contents of this chapter is published in **Kyungpook Mathematical Journal, 54(4) (2014), 629-638(Korea).**

In **chapter four**, we introduce ordered (generalized) (m, n)- Γ-ideals in ordered Γ-semigroups. Then we characterize ordered Γ-semigroup through ordered (generalized) (0, 2)-Γ-ideals, ordered (generalized) (1, 2)- Γ-ideals and ordered (generalized) 0-minimal (0, 2)-Γ-ideals. Also, we investigate the notion of ordered (generalized) (0, 2)-bi-Γ-ideals, ordered 0-(0, 2) bisimple Γ-semigroups and ordered 0-minimal (generalized) (0, 2)-bi-Γ-ideals in ordered Γ-semigroups. The entire contents of this chapter is published in **Hacettepe Journal of Mathematics and Statistics 44(2) (2015), 247-254(Turkey).**

In **chapter five**, we use the notion of quasi- and bi-ideals to study some properties of locally associative LA-Γ-semigroups, anti-rectangular LA-Γ-semigroups, (m, n) simple LA-Γ-semigroups, minimal bi-Γ-ideal, semiprime Γ-ideal and quasi-regular LA-Γ-semigroups. The entire contents of this chapter is published in **Southeast Asian Bulletin of Mathematics 39(1) (2015), 1-12(China).**

Chapter six deals with quasi-ideals of regular, intra-regular ordered Γ-semigroups and of abstract affine Γ-nearrings. In the first section of the chapter six, we introduce the notion of ordered quasi-Γ-ideals of regular ordered Γ-semigroups and study some basic properties of ordered quasi-Γ-ideals of ordered Γ-semigroups. We also characterize regular ordered Γ-semigroups in terms of its ordered quasi-Γ-ideals, ordered right Γ-ideals and ordered left Γ-ideals. The entire contents of the section two are published in **Algebra, Volume (2013), Article ID 565848, 7 pages, doi:10.1155/2013/565848 (USA).**

In the **second section** of chapter six, we characterize intra-regular ordered Γ-semigroups. We investigate ordered Γ-semigroups in which the regular and intra-regular sub-Γ-semigroups are idempotents, left regular idempotents, commutative and idempotents. Then we characterize intra-regular ordered Γ-semigroups through ordered bi-Γ-ideals and ordered quasi-Γ-ideals, ordered bi-Γ-ideals and ordered right Γ-ideals, ordered bi-Γ-ideals and ordered left Γ-ideals of ordered Γ-semigroups. The entire contents of this section is published in **International Journal of Pure and Applied Mathematics, 98(2) (2015), 221-230(Bulgaria).**

The **last section of chapter six** is devoted to the study of abstract affine Γ-nearrings. Here, we obtain an equivalent condition for a subgroup of $(M, +)$ to become a quasi-ideal of Γ-nearring M. Finally, we present an example of quasi-ideal Q of a Γ-nearring M which is neither a left ideal nor a right ideal of M. The entire contents of the third section is published in **International Journal of Pure and Applied Mathematical Sciences, 7(1) (2014), 43-49(India).**

In the end, a comprehensive bibliography with the author's name in alphabetical order is given enlisting books and papers which have been referred to in the thesis.