

**Name of the Research Scholar:** IFTIKHAR

**Name of the Supervisor:** Prof. Musheer Ahmad

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### **ABSTRACT**

In the real world situations fuzzy set theory (proposed by Zadeh [1965]) plays a prominent role with the wide applications in many disciplines. This provides a sufficient motivation to researchers to review various concepts and results based on uncertainty in the broader environments of fuzzy set theory. In many circumstances the obtained solutions using crisp concept is meaningless. This shortcoming has removed by introducing fuzzy set theory. In general obtained solutions using fuzzy data is approximate rather than exact but it converges to the exact solution. Therefore most of the concepts in the field of mathematics, engineering, medical and economics can be redefined using fuzzy information. Hence it is an essential need to popularize these ideas based on fuzzy concepts..

The first chapter is introductory, where we have summarized some definitions and results based on fuzzy set theory.

The main objective of second chapter is to study the Jacobi and Gauss-Seidel numerical schemes in the matrix form to solve a fuzzy system of linear equations  $Ax = y$ . The obtained solutions are compared with the exact solutions obtained by Friedman *et.al.* [48]. It can be concluded that Gauss-Seidel gives better approximations than Jacobi numerical scheme into matrix form. Numerical examples are solved for the support of proposed method.

The aim of third chapter is to propose a new methodology and approach to solve a hyperbolic one dimensional telegraph equation with Dirichlet's boundary conditions using fuzzy membership functions and alpha-cuts at different grid points as per finite difference method. The present approach for the numerical solution based on

fuzzification technique is an efficient and computationally fast algorithm. It can be concluded that the proposed approach can easily deal with more complex problems of ordinary and partial differential equations. The obtained numerical results are very close to the exact solutions and also with the earlier studies.

In the fourth chapter the maximum age-group of cigarette smokers who have started smoking due to various reasons have been investigated. The authors have divided and defined four types of matrices which are called initial raw data matrix (IRD matrix), average time dependent data matrix (ATD matrix), refined time dependent data matrix (RTD matrix), and combined effect time dependent data matrix (CETD matrix), and apply an algebraic applications of fuzzy matrix theory to deal with smoking problem.

In the fifth chapter certain approximations are assumed by considering symmetric fuzzy decision variables in order to solve a fully fuzzy linear programming problem (FFLPP). These assumptions have been reduced number of linear equations in the crisp linear programming problem corresponding to the given fully fuzzy linear programming problem. The optimal solutions and optimal values so obtained are very close to the exact solutions and exact optimal values.

In chapter six a new approach is proposed to dealing with the tie situations. To break the tie situations fuzzy analytical hierarchy process (fuzzy AHP) is applied by assuming triangular fuzzy numbers for the pair-wise comparisons. Then extent analysis method (EAM) [26] is used for deriving the fuzzy synthetic extent values and normalized weight vectors. The working procedure is illustrated with the help of solved numerical examples.

In chapter seven a special class of fuzzy pair-wise comparison matrices is introduced using triangular fuzzy numbers and a fixed common ratio  $0 < r \leq 1$  among their tuples. Afterward, an innovative method is proposed for deriving fuzzy priority vectors. It can be concluded that proposed method for fuzzy prioritization is an efficient and computationally fast algorithm and it is capable to reduce the size of computational work.