Abstract

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Title: Relevance of Hypergeometric functions as a powerful tool to address Mathematical issues
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In this thesis, we obtain analytical solutions and generalizations of some infinite Fourier sine and cosine transforms based on Ramanujan integrals [*Mess. Math.*, XLIV, 75-86, 1915]. The integrals are calculated in terms of ordinary hypergeometric functions ${}_2F_3$ and Meijer's G-function, with suitable convergence conditions. Moreover as applications of these integrals, new infinite summation formulae are obtained. We have described some infinite Fourier sine and cosine transforms of Ramanujan. Thus certain Ramanujan integrals, which may be different from those presented here, can also be evaluated in a similar way. The results established are thus significant in larger context. We note that various new results and applications can be obtained from our general theorems by appropriate choice of parameters. We have obtained some definite integrals containing the quotients of hyperbolic functions, in terms of hypergeometric functions, Digamma functions, Lower Beta function of one variable, and Gamma function. Also, Laplace transforms have been obtained of an arbitrary power of some finite series containing hyperbolic sine and cosine functions. We have also calculated Laplace transforms of positive integral powers of sine and cosine functions, in terms of Gauss hypergeometric and Beta functions. Thus certain integrals of hyperbolic and trigonometric functions, which may be different from those presented here, may also be evaluated by the hypergeometric approach. We have analysed numerically and graphically three doubtful theorems of Srinivasa Ramanujan, stated by him without proof [*Mess.Math.*, XLIV, pp. 75-86, 1915]. The analysis is done by using Wolfram Mathematica and Matlab softwares. Here we have

concluded that the same three theorems of Srinivasa Ramanujan edited by Hardy- Aiyar-Wilson (without giving any analytical proofs) are indeed correct [see Collected Papers of Srinivasa Ramanujan, Published by Cambridge University Press 1927]. Finally, we have studied extensions of Srivastava's triple hypergeometric functions in view of extended Beta functions. Also, we give some of its main properties, namely the Mellin transform, a differential formula, recursion formulas and a bounded inequality. It is expected that these extensions will find applications in various branches of applied mathematics as well as mathematical physics. Other possible extensions of Srivastava triple hypergeometric functions are under investigation.