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FINDINGS

In the present work, an attempt has been made to study the **CONTRIBUTION TO THE STUDY OF HYPERGEOMETRIC FUNCTIONS WITH APPLICATIONS**. My thesis is spread over in **Ten Chapters**. A brief summary of the problems is presented at the beginning of each chapter and then each the chapters is divided into a number of sections. Equations in every section are numbered separately.

The aim of the **First Chapter** is to introduce several classes of special functions, which occur rather more frequently in the study of summations and transformations needed for the presentation of subsequent chapters. In this chapter, we have discussed Gamma functions, Pochhammer symbols and Associated results; Ordinary hypergeometric function and its generalization; Truncated and terminating generalized hypergeometric series; Appell's double hypergeometric functions; Kampé de Fériet double hypergeometric function; Decomposition of ${}_pF_q$; Series rearrangement techniques ; Generating functions and generating relations; Orthogonal polynomials; Some transformations and summation theorems; Some well known results of fractional calculus and hypergeometric forms of some mathematical functions; Some basic results for covers(x), covers$^{-1}(x)$, vers(x) and vers$^{-1}(x)$.

It provides a systematic introduction to most of the important special functions that commonly arise in practice and explores many of their silent properties. This chapter is also intended to make the thesis as much self contained as possible.

In **Second Chapter**, we obtain hypergeometric forms of some functions via differential equation approach by changing the independent and dependent variables inthe suitable ordinary differential equations of first and second order, and comparing the resulting ordinary differential equations with standard ordinary differential of Leibnitz and Gauss. Also, we obtain hypergeometric representation of certain inverse trigonometric functions using series iteration techniques with summation theorems for terminating ${}_5F_4(1)$ and Pfaff-Kummer linear transformation. Further by making applications of hypergeometric forms of arcsine and arctangent functions, we establish successive differentiation and successive integration of these inverse trigonometric functions.

In **Third Chapter**, we have established, hypergeometric forms of some typical trigonometric functions with suitable convergence conditions and also we have given their detailed derivations by using Taylor's expansion, Maclaurin's expansion, Decomposition formula, Leibnitz theorem and Clausen's formula. Further we have given nth differential coefficient of some hypergeometric forms using series rearrangement techniques.

In **Fourth Chapter**, we aim at obtaining the semi-differentials of complete Ellipticintegrals of different kinds in terms of algebraic functions by using Euler's linear transformation, Pfaff-Kümmer linear transformation and series manipulation techniques. We also obtain semi-differentials of some elementary functions and some semi-differentials of complete elliptic integrals of different kinds. Further we obtain some semi-differentials of difference of two complete elliptic integrals of different kinds.

In **Fifth Chapter**, we have established the closed forms of some reduction formulas for Gauss and Clausen hypergeometric functions using the series rearrangement techniques and prove them. Also, we establish difference of two divergent Gauss series having positive unit argument is convergent and also we derive summation of some infinite series.

In **Sixth Chapter**, we establish a summation theorem for a terminating ${}_4F_3$ series with positive unit argument by using a series rearrangement technique. Again , we derive a reduction formula for Kampé de Fériet double hypergeometric function and general double series identity respectively by using our main summation theorem. Further we obtain hypergeometric forms of quadratic polynomials. Finally we obtain hypergeometric form for cubic polynomials and biquadratic polynomials.

In **Seventh Chapter**, this chapter mainly concerns with three theorems involving generating functions expressed in terms of single and double Laplace and Beta Integrals. These theorems have been applied to obtain bilinear and bilateral generating functions involving polynomials of generalized Rice, Jacobi, Lagrange, Gegenbauer and other polynomials hypergeometric in nature. A number of generating functions have also been obtained as special cases. One variable special cases of hypergeometric polynomials are important in several applied problems. Further we also have established another three theorems expressed in terms of generalized hypergeometric function ${}_AF_B[x]$.Theorem 1 establishes a generating function for generalized hypergeometric ${}_{A+1}F_{B+1}[x]$ series expressed in terms of beta transform of one variable. Theorem 2 is associated with obtaining a generating function for ${}_{A+1}F_B[x]$ series expressed in terms of Laplace transform of one variable. Theorem 3 is devoted to proving a generating function for ${}_{A+2}F_B[x]$ series expressed in terms of Laplace transform of two variables. We also have made applications of the these Theorem for obtaining bilinear and bilateral generating functions of hypergeometric polynomials.

In **Eighth Chapter**, This chapter contains mainly three theorems involving Kampé de Fériet's function $F^{(2)}$ and expressed in terms of single and double Laplace and Beta integrals. The theorems, in turn, yield, as special cases, a number of linear, bi- linear and bilateral generating functions of generalized polynomials of Rice, Jacobi polynomials, ultraspherical, generalized Laguerre, Bedient polynomials and other polynomials hypergeometric in nature. One variable special cases of generalized polynomials are useful in several applied problems.Theorem 1 establishes a generating function for Kampé de Fériet's double series, expressed in terms of beta transform of one variable. Theorem 2 is associated with obtaining a generating function involving Kampé de Fériet's double series $F^{(2)}$, expressed in terms of Laplace transform of one variable. Theorem 3 is devoted to proving a generating function for $F^{(2)}$ series , expressed in terms of Laplace trans- form of twovariable.

In Ninth Chapter, we provide the exact expressions (not found in the literature) for the curved surface area of revolution (about x-axis and y-axis) of horizental andoblique hyperbolas. The closed form expressions for the curved surface area are obtained in terms of Gauss function, Clausen function and Appell's hypergeometric functions of first kind.

We also have obtained exact expressions for curved surface area of a three dimensional surface (obtained by revolving the arc of hyperbolas about co-ordinate axes) using hypergeometric function approach. Any values of parameters and arguments leading to the results which do not make sense are tacitly excluded

In Tenth Chapter , we have concluded the thesis and also discussed the future scope

At the end of the thesis, a detailed bibliography appears with the authors namein alphabetical order. References to the bibliography are numbered. The thesis include with appendices which contains reprints of few published papers and certificates of